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On the shape of the auroral zones

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With 4 figures in the text

ABSTRACT

Model experiments on Alfvén's theory of aurorae, performed earlier by the author, resulted in auroral zones with a smaller eccentricity than theoretically expected if space charges are neglected. Preliminary measurements of the charges in the model have now been made, and on the basis of this it is possible to theoretically account for the smaller eccentricity. From the theory it also comes out that the measured space charge distribution will tend to increase the polar distance of the auroral zones. The influence of variable terrella voltage has also been investigated and the result is a good agreement between theory and experiment.

Introduction

Model experiments on Alfvén's theory of aurorae (Alfvén, 1950) have been performed earlier by Block (1955). These resulted in a lower eccentricity than indicated by the theory, which neglects space charges. It has been suggested that space charges may account for the difference (Block, 1956). Another suggestion was that the difference may partly be accounted for by the fact that the terrella potential in the experiments differed considerably from that assumed in the theory (Block, 1955). This paper presents some results of measurements of space charges and of the influence of the terrella potential together with theoretical calculations which largely agree with the experimental results.

Experiments

Measurements of the space charges in the model have been attempted by G. Apelqvist and C. G. Fälthammar (unpublished). They put probes in the discharge region (dark discharge) and measured the potential distribution. Their measurements could not be fulfilled because of an accident with the apparatus, so we do not know the potential in very much detail. However, sufficient material is available to get a preliminary rough picture of the charge distribution in the equatorial plane. Since this agrees with what we should qualitatively expect we publish it here with the reservation that more complete measurements will modify it to some extent.

The result is shown in Fig 1. The upper part of the figure shows the difference, ΔV, between the potential along the x-axis in the model when there is a discharge

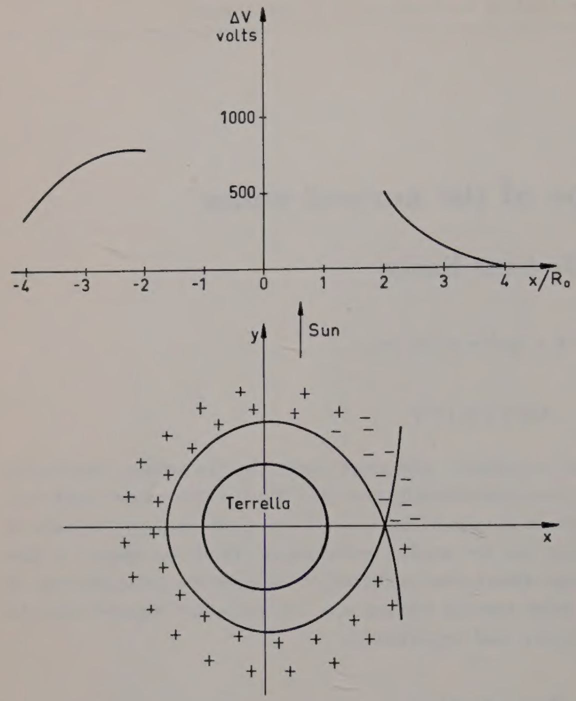

Fig. 1. Space charge distribution in the equatorial plane of the experimental discharge and its influence on the potential along the x-axis, which is directed in the 6-hour direction through the earth's centre. $V_a/V_d = 0.5$ which gives no net current to the terrella.

and the potential when there is no discharge but unchanged potentials on all electrodes, i.e. the potential due to space charges as a function of x. It is seen that ΔV is always positive but less positive on the cathode side of the terrella than on the anode side. The lower part of the figure shows the sign of the space charges in the equatorial plane. The distribution of charges differs from that in Alfvén's auroral theory, because it is turned 90° to the "east" so that we have negative space charge on the cathode side. However, this agrees with what should be expected, because as pointed out by Block (1956) the magnetic field is too weak to make the radius of curvature of the ion orbits small in comparison to the forbidden zone, so the ions move essentially along the electric field. Hence the anode side of the terrella acts as a collector of ions and the cathode side as an anode, collecting electrons. The total current to the terrella is zero, but there is a current from the anode side to the cathode side (also observed in the experiments), corresponding to a current from the evening side to the morning side in the auroral zones of the earth as explained in an earlier paper (Block, 1956, p. 317). According to the theory of probes in discharges, there should be positive space charge outside a probe collecting ions, and negative space charge outside a probe collecting electrons. Thus, if the terrella is regarded as a sort of double probe the above picture agrees with probe theory.

The influence of the terrella potential on the location of the auroral zones has been investigated in the dark discharge. The potential difference, V_d, between anode and cathode was kept constant, $V_d = 5$ kV, the magnetic field at the poles of the terrella $B_p = 1000$ gauss, the pressure $p = 8 \times 10^{-4}$ mm Hg and the discharge current

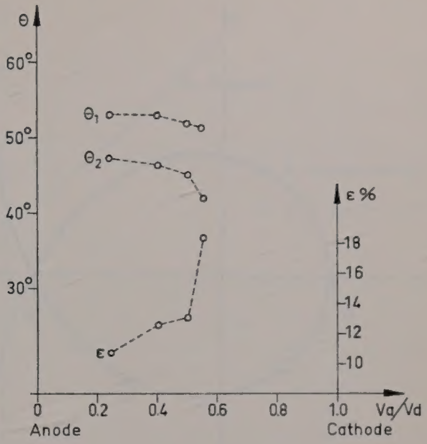

Fig. 2. The eccentricity ε, the maximum polar angle θ_1 and the minimum polar angle θ_2 as functions of the terrella voltage in the experiments.

$I_d = 0.6$ mA. The potential, V_a, of the anode relative to the terrella was varied. The result of the measurements is shown in Fig. 2. θ_1 is the maximum angular polar distance and θ_2 is the minimum polar distance of the auroral zones. If $V_a/V_d = 0$ the terrella is at anode potential, and if $V_a/V_d = 1$ it is at cathode potential.

It is seen from Fig. 2 that the more V_a/V_d increases, the closer do the auroral zones come to the poles and the larger is the eccentricity.

Theory of the influence of the terrella potential

The equations of the electron orbits in a magnetic dipole field with the dipole in the $-z$-direction in the presence also of a homogeneous electric field E_0 in the x-direction and a homogeneous magnetic field B_0 in the z-direction has been obtained by Alfvén (loc. cit.). If the terrella is charged, however, the electric field is not homogeneous any more, and then Alfvén's equations are modified in the following way where the symbols have their usual meaning (cf. Alfvén, 1950, p. 179, and Block, 1955, p. 85).

$$V = V_0 + E_0(x_D - x) + Q/R, \tag{1}$$

$$B = B_0 + b/R^3, \tag{2}$$

$$\mu = eV/B. \tag{3}$$

Q is a constant proportional to the charge of the terrella. The terrella has been assumed to be infinitely small so that the charge can be regarded as a point charge. However, in order to make comparisons with the experiments we must assume a finite terrella radius R_0, in which case the terrella potential in relation to the mean of the anode and cathode potential is

$$V_t = Q/R_0. \tag{4}$$

Combining (1), (2) and (3) we find

$$x_D - x = L^4/R^3 - K/R, \tag{5}$$

$$L^4 = \mu b/eE_0; \qquad K = Q/E_0 = V_t R_0/E_0. \tag{6}$$

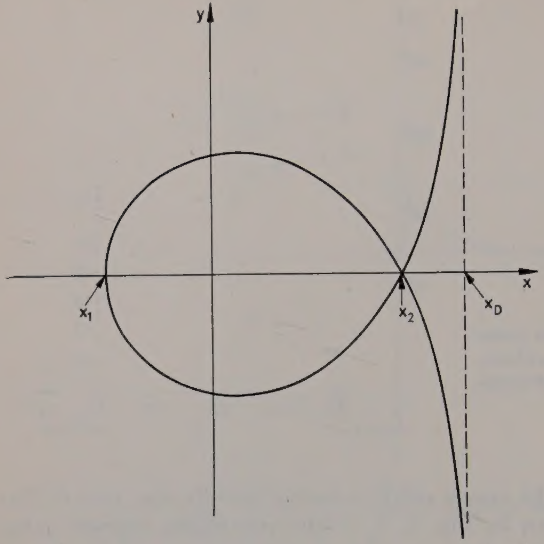

Fig. 3. The forbidden zone curve in the equatorial plane.

If $Q=0$ we have

$$x_D = \tfrac{4}{3}\sqrt[4]{3}\,L = 1.76\,L,$$

$$x_2 = \sqrt[4]{3}\,L = 1.32\,L \tag{7}$$

$$x_1 = -0.74\,L.$$

For the definition of x_1 and x_2 see Fig. 3. If $Q \neq 0$ we find

$$x_2 = \sqrt{-\tfrac{1}{2}K + \sqrt{3L^4 + \tfrac{1}{4}K^2}} \tag{8}$$

with

$$x_D = x_2 + L^4/x_2^3 - K/x_2 \tag{9}$$

and x_1 is determined by

$$x_D - x_1 = K/x_1 - L^4/x_1^3, \tag{10}$$

In Fig. 4 the results of (8) and (10) have been plotted as a function of V_a/V_d, where according to (4) and (6) V_a/V_d is determined by

$$V_a/V_d = 0.5 - V_t/V_d = 0.5 - K E_0/R_0 V_d. \tag{11}$$

For reasons which will become clear in the next paragraph we have chosen $L = 2.20\ R_0$ when calculating the theoretical curves in Fig. 4. (cf. eq. (27)). As a comparison the experimental values of x_1 and x_2, obtained by projection of the experimental auroral zone along the dipole lines, are shown in the figure. It is seen that theory and experiment agree, in as much as that they both indicate increasing x_1 and x_2 with increasing V_a/V_d, i.e. decreasing terrella potential. The rate of increase of x_1 and x_2 respectively are also about the same theoretically and experimentally.

If we had chosen $L < 2.20\ R_0$ the theoretical curves would have been lowered correspondingly but as will be seen in the next praragraph it is possible to account for the discrepancy between theory and experiment by considering the influence of space charges.

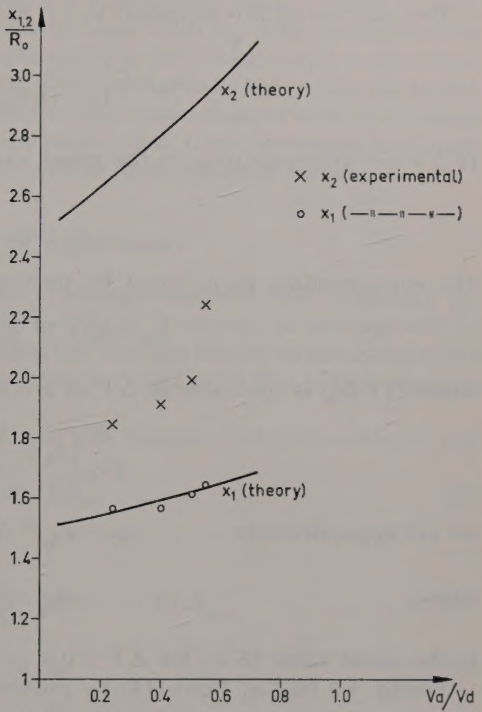

Fig. 4. The distances x_1 and x_2 as functions of the terrella voltage according to the theory neglecting space charges. The rings and crosses indicate experimental values.

Theory of the influence of space charges

We assume that the space charges change the potential by

$$\Delta V = \Delta V(x,y,z), \tag{12}$$

where $\Delta V \to 0$ at infinity. This means that the equations of the electron orbits are

$$V = V_0 + E_0(x_D - x) + \Delta V, \tag{13}$$

together with (2) and (3). A combination of these gives

$$x_D - x = L^4/R^3 - \Delta V/E_0. \tag{14}$$

Differentiation with respect to x gives

$$-1 = -\frac{3L^4}{R^5}(x + yy') - \frac{1}{E_0}\left\{\frac{\partial \Delta V}{\partial x} + \frac{\partial \Delta V}{\partial y}y'\right\}, \tag{15}$$

where $R^2 = x^2 + y^2$.

Putting $y = 0$, $yy' = 0$ and $(\partial \Delta V/\partial y)y' = 0$, we obtain

$$1 = \frac{3L^4}{x^4} + \frac{1}{E_0}\frac{\partial \Delta V}{\partial x}. \tag{16}$$

The solution of this equation is $x = x_2$,

$$x_2 = L \left\{ \frac{3}{1 - (1/E_0)(\partial \Delta V / \partial x)} \right\}^{\frac{1}{4}}_{x = x_2}. \tag{17}$$

If $\Delta V \equiv 0$ this goes over in the usual value of

$$x_2 = x_{20} = \sqrt[4]{3}\, L.$$

The corresponding x_D is found by putting $x = x_2$, $y = 0$ in (14)

$$x_D = x_2 + L^4/x_2^3 - \Delta V(x_2)/E_0, \tag{18}$$

where $\Delta V(x_2)$ is the value of ΔV at $x = x_2$, $y = z = 0$. Since

$$\frac{d}{d x_2} \left(x_2 + \frac{L^4}{x_2^3} \right)_{x_2 = x_{20}} = 0,$$

we get approximately

$$x_D = x_{D_0} - \Delta V(x_2)/E_0, \tag{19}$$

where

$$x_{D_0} = \tfrac{4}{3} \sqrt[4]{3}\, L$$

is the usual value of x_D for $\Delta V = 0$.

Finally we find x_1 from (14) by putting $R = -x_1$, $x = x_1$:

$$x_1 = x_D + L^4/x_1^3 + \Delta V(x_1)/E_0, \tag{20}$$

or using (19)

$$x_1 = x_{D_0} + \frac{L^4}{x_1^3} + \frac{1}{E_0} \{\Delta V(x_1) - \Delta V(x_2)\}. \tag{20 a}$$

Introducing

$$D = \frac{1}{E_0} \{\Delta V(x_1) - \Delta V(x_2)\} \tag{21}$$

we find from (20a)

$$\partial D / \partial x_1 = 1 + 3 L^4 / x_1^4, \tag{22}$$

which gives an approximate solution of (20a)

$$x_1 = x_{10} + \frac{D}{1 + 3 L^4 / x_{10}^4}. \tag{23}$$

Now the usual value of x_1 for $\Delta V \equiv 0$ is

$$x_{10} = -0.74\, L.$$

Introducing this in (23) we get

$$x_1 = x_{10} + \frac{D}{11} = x_{10} + \frac{1}{11 E_0} \{\Delta V(x_1) - \Delta V(x_2)\}. \tag{24}$$

According to the experimental curve in Fig. 1 ΔV is always positive but $\Delta V(x_1)$ is slightly larger than $\Delta V(x_2)$ so that x_1 is slightly larger than x_{10}. This means that the auroral zone is pushed somewhat towards the equator here. At $x = x_2$ it is seen from (17) that the same happens if $\partial \Delta V / \partial x < 0$. This appears to be the case in the model.

Qualitatively the results of this treatment is that space charges seem to increase the polar distance of the auroral zone. This means that L can be somewhat larger and the beam fields E_0 and B_0 accordingly smaller than assumed earlier.

Application to the experiments

A numerical application of these results to the experiments is very uncertain because of the still insufficient potential data available. However, an attempt will be made in order to see what is needed to explain the well established experimental data given in Fig. 4. The only potentials we need then is $\Delta V(x_1) - \Delta V(x_2)$. This seems to be of the order of 100 volts.

The total discharge voltage was 5 kV and the distance between cathode and anode is 11 R_0. Hence

$$E_0 = 5000/11\,R_0. \tag{25}$$

Then (24) and Fig. 4 ($V_a/V_d = 0.5$) gives

$$x_1 = -0.74\,L + \frac{R_0}{5000} \cdot 100 = -1.61\,R_0. \tag{26}$$

From this we get

$$L = 2.20\,R_0. \tag{27}$$

Fig. 4 shows that the experimental $x_2 = 2.0\,R_0$.

Equation (17) gives

$$(\partial \Delta V / \partial x)_{x=x_2} = E_0 (1 - 3\,L^4/x_2^4). \tag{28}$$

Our numerical values give

$$\left(\frac{\partial \Delta V}{\partial x}\right)_{x=x_2} = \frac{5000}{11\,R_0}\left(1 - \frac{3\,(2.2)^4}{2^4}\right) = -\frac{1550}{R_0}. \tag{29}$$

This means a field strength due to the space charges of about 300 V/cm in addition to E_0 which is only 85 V/cm. In view of the curves shown in Fig. 1 this does not seem unreasonable.

Measurements of the total current to the terrella revealed that this was zero when $V_a/V_d = 0.5$. Thus if V_t is more negative the terrella mainly receives ions and the electrons are reflected away. It was also noticed that if $V_a/V_d > 0.55$ a glow discharge was initiated. This may be due to intense ionization by oscillating electrons, reflected by the negative terrella potential. In any case the space charge should be radically changed when we approach this state. This may account for the sharp bend of the experimental curve for x_2 shown in Fig. 4 and, which is the same, the strong increase of the eccentricity at $V_a/V_d > 0.5$ in Fig. 2. However, it may also be that the bend of the curve is not quite as sharp as shown in the figure, because the experimental errors in ε may be as large as one tenth of the measured values. According to our theory the eccentricity should be very insensitive to the terrella voltage if space charges are neglected.

From the experimental curves in Fig. 4 we find for $V_a/V_d = 0.5$

$$\tfrac{1}{2}(x_1 + x_2) = 1.80\ R_0.$$

According to the simplified theory which neglects space charges we have

$$\tfrac{1}{2}(x_1 + x_2) = 1.03\ L.$$

This means that the simple theory gives the same auroral mean latitudes as our experiments if

$$L = 1.75\ R_0.$$

However, when we accounted for space charges we found in eq. (27) that $L = 2{,}20\ R_0$. Hence, the more complete theory can give the same auroral mean latitudes as the simple theory provided L is increased by a factor

$$k = 2.20/1.75 = 1.26.$$

Since $L \sim E_0^{-0.25}$ the beam fields E_0 and B_0 may be only 40 % of what has been assumed before, and still produce aurorae at the same latitudes, if these experimental results are applicable also to the cosmical phenomena. This is, however, uncertain because the space charge distribution in nature is different from that in the model due to the relative difference in the radius of curvature of the ion orbits. On the other hand, it may be that this does not matter so much if only the electric field at x_2 is strongly increased by space charges in nature. This would increase the drift velocity due to the electric field so that the point x_2, where the magnetic gradient drift balances the electric field drift, is pushed closer to the earth.

These results indicate very clearly that Alfvén's auroral theory is applicable to the model experiments.

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